Int. J. Heat Mass Transfer. Vol. 18, pp. 811–812. Pergamon Press 1975. Printed in Great Britain

ESTIMATION OF DIFFUSIONAL THERMOEFFECT CONTRIBUTION TO HEAT TRANSFER IN GAS MIXTURES IN ELECTRIC FIELD (STEADY STATE)

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(*Received* 25 *December* 1974)

Abstract—The methods to estimate the contribution of the diffusional thermoeffect to total heat transfer in a gas mixture in the electric field are given. Consideration is made of the case of a steady state.

NOMENCLATURE

α_T, thermodiffusional constant;
k_T, thermodiffusional ratio;
λ_0, thermal conductivity of a mixture with uniform concentration;
λ_∞, thermal conductivity of a mixture in steady state;
$\tilde{\lambda}_\infty$, thermal conductivity of a mixture in the presence of an electric field;
λ_{D^T}, diffusional thermoeffect contribution to thermal conductivity of a mixture;
$\tilde{\lambda}_{D^T}$, diffusional thermoeffect contribution to thermal conductivity of a mixture in the presence of an electric field;
D_{12}, mutual diffusion coefficient;
P, pressure;
T, temperature;
x_1, concentration of a light component;
n_1, numerical density of a light component;
n, total numerical density;
$\bar{\mathbf{v}}$, diffusional molecular velocity of 1st component;
$\tilde{\bar{\mathbf{v}}}_1$, diffusional molecular velocity of 1st component in the presence of an electric field;
q, heat flux;
$\tilde{q}$, heat flux in the presence of an electric field;
D_1^T, thermodiffusion coefficient.

CONTRIBUTION of the diffusional thermoeffect to heat conduction without an electric field in steady systems with a temperature gradient is defined by [1]:

$$\lambda_{D^T} = \lambda_0 - \lambda_\infty = \frac{PD_{12}}{T}\alpha_T k_T. \tag{1}$$

By analogy with equation (1) contribution of the diffusional thermoeffect to heat transfer in gas mixtures in the presence of an electric field is defined by:

$$\tilde{\lambda}_{D^T} = \lambda_0 - \tilde{\lambda}_\infty. \tag{2}$$

If an electric field is imposed on a gas mixture, then its effect will be manifested by changes in the diffusional velocity

$$\tilde{\bar{\mathbf{v}}}_i = \frac{n^2}{n_i\rho}\sum_j m_j D_{ij}\tilde{\mathbf{d}}_i - \frac{1}{n_i m_i}D_i^T\frac{\partial \ln T}{\partial \mathbf{r}}, \tag{3}$$

where

$$\tilde{\mathbf{d}}_i = \mathbf{d}_i - \frac{n_i m_i}{P\rho}\left(\frac{\rho}{m_i}\mathbf{x}_i - \sum_i n_i \mathbf{x}_j\right), \tag{4}$$

and $\mathbf{x}_i$ and $\mathbf{x}_j$ are the external forces due to the electric field which influence molecules of the ith and jth species; the second term in relation (4) is the contribution of the electric field to the component concentration vector.

Re-write relation (3) in the form:

$$\tilde{\bar{\mathbf{v}}}_i = \bar{\mathbf{v}}_i + \frac{n^2}{n_i\rho}\sum_j m_i D_{ij}\frac{n_i}{n}\left\{\frac{n_i}{\rho}\mathbf{x}_i - \frac{n_i m_i}{P\rho}\sum_i n_i \mathbf{x}_j\right\}$$
$$= \bar{\mathbf{v}}_i + \bar{\mathbf{v}}_i^*,$$

where

$$\bar{\mathbf{v}}_i^* = \frac{n^2}{n_i\rho}\sum_j m_i D_{ij}\frac{n_i}{n}\left\{\frac{n_i}{P}\mathbf{x}_i - \frac{n_i m_i}{P\rho}\sum_i n_i \mathbf{x}_j\right\}. \tag{5}$$

It may be shown that the heat flux density vector in the presence of an electric field is defined by:

$$\tilde{\mathbf{q}} = \mathbf{q} + \tfrac{5}{2}kT\sum_i n_i\bar{\mathbf{v}}_i^* + \frac{kT}{n}\sum_{ij}\frac{n_j D_i^T}{m_i D_{ij}}(\bar{\mathbf{v}}_i^* - \bar{\mathbf{v}}_j^*), \tag{6}$$

where

$$\mathbf{q} = -\lambda_0\frac{\partial T}{\partial \mathbf{r}} + \tfrac{5}{2}kT\sum_i n_i\bar{\mathbf{v}}_i + \frac{kT}{n}\sum_{ij}\frac{n_j D_i^T}{m_i D_{ij}}(\bar{\mathbf{v}}_i - \bar{\mathbf{v}}_j)$$

is the heat flux density vector with no electric field. For a binary gas mixture of polar (subscript 1) and non-polar (subscript 2) molecules, $\mathbf{x}_1 = \text{grad}\,\mu E$, $\mathbf{x}_2 = 0$* where μ is the dipole moment, E is the electric field strength, the expression for the vector of the heat flux density in the coordinate system moving with a mean numerical velocity takes the form:

$$\tilde{\mathbf{q}} = \mathbf{q} + \frac{x_2^2 x_1 k_T m_1 m_2^2 P D_{12}}{kT(m_1 x_1 + m_2 x_2)^3}\mathbf{x}_1 \tag{7}$$

Since the mass flow in a steady state is zero, then

$$\tilde{\mathcal{J}}_1 = n_1 m_1 \tilde{\bar{\mathbf{v}}}_1 = \frac{n^2}{\rho}m_1 m_2 D_{12}\tilde{\mathbf{d}}_1 - D_1^T\frac{\partial \ln T}{\partial \mathbf{r}} = 0.$$

*Experiments on thermal conductivity of mixtures in an electric field are carried out just for the case considered [2–3].

Hence

$$-\frac{n^2}{\rho}m_1 m_2 D_{12}\frac{n_1 kT}{Px_1}\frac{\partial x_1}{\partial \mathbf{r}}-\frac{n^2}{\rho}\frac{m_1 m_2 D_{12} n_1}{P}\mathbf{x}_1 - D_1^T\frac{1}{T}\frac{\partial T}{\partial \mathbf{r}} = 0$$

and

$$\mathbf{x}_1 = -\left(\frac{kT}{x_1}\frac{\mathrm{d}x_1}{\mathrm{d}T}+\frac{Pk_T}{n_1 T}\right)\frac{\partial T}{\partial \mathbf{r}}. \tag{8}$$

With regard for expression (8) relation (7) assumes the form

$$\tilde{\mathbf{q}} = \mathbf{q}-\frac{PD_{12}x_2^2 m_1 m_2^2 k_T^2}{(m_1 x_1+m_2 x_2)^3}\left[\frac{1}{k_T}\frac{\mathrm{d}x_1}{\mathrm{d}T}+\frac{1}{T}\right]\frac{\partial T}{\partial \mathbf{r}}$$

$$= -\left[\lambda_0-\lambda_{D^T}+\frac{PD_{12}x_2^3 m_1 m_2^2 k_T^2}{(m_1 x_1+m_2 x_2)^3}\left(\frac{1}{k_T}\frac{\mathrm{d}x_1}{\mathrm{d}T}+\frac{1}{T}\right)\right]\frac{\partial T}{\partial \mathbf{r}}$$

$$= -\tilde{\lambda}_\infty\frac{\partial T}{\partial \mathbf{r}}. \tag{9}$$

It may be shown that

$$\tilde{\lambda}_{D^T} = \lambda_0-\tilde{\lambda}_\infty = \lambda_{D^T}\left[1+C_1 C_2^3\left(1+\frac{T}{k_T}\frac{\mathrm{d}x_1}{\mathrm{d}T}\right)\right]. \tag{10}$$

where $C_i = (x_i m_i)/(x_i m_i + x_j m_j)$ is the mass concentration of the ith species.

With no electric field,

$$\frac{\mathrm{d}x_1}{\mathrm{d}T} = -\frac{k_T}{T} \quad \text{and} \quad \tilde{\lambda}_{D^T} = \lambda_{D^T}.$$

Since $3kT = \mu E$, then

$$\mathbf{x}_1 = \operatorname{grad}\mu E = 3k\operatorname{grad} T. \tag{11}$$

With the aid of expressions (8) and (11) we have:

$$\frac{\mathrm{d}x_1}{\mathrm{d}T} = -\frac{x_1}{T}(3-k_T) \tag{12}$$

and the contribution of the diffusional thermoeffect to heat transfer in gas mixtures in an electric field (steady state) is estimated by:

$$\tilde{\lambda}_{D^T} = \lambda_{D^T}\left[1-C_1 C_2^3\left\{1-\frac{x_1}{k_T}(3-k_T)\right\}\right]. \tag{13}$$

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ESTIMATION DE LA CONTRIBUTION DE L'EFFET THERMODIFFUSIONNEL AU TRANSFERT DE CHALEUR EN MELANGE GAZEUX DANS UN CHAMP ELECTRIQUE (ETAT STATIONNAIRE)

Résumé—Les méthodes pour estimer la contribution de l'effet thermo-diffusionnel au transfert de chaleur global, sont données pour un mélange de gaz dans un champ électrique. On considère le cas de l'état stationnaire.

ABSCHÄTZUNG DES BEITRAGES DES DIFFUSIONALEN THERMOEFFEKTES ZUM WÄRMETRANSPORT IN GASMISCHUNGEN IM ELEKTRISCHEN FELD

Zusammenfassung—Die Methoden zur Abschätzung des Beitrags des diffusionalen Thermoeffekts zum gesamten Wärmetransport in einer Gasmischung im elektrischen Feld werden angegeben. Die Betrachtungen gehen vom Fall eines stabilen Zustandes aus.

ОЦЕНКА ВКЛАДА ДИФФУЗИОННОГО ТЕРМОЭФФЕКТА В ПЕРЕНОС ТЕПЛА В ГАЗОВЫХ СМЕСЯХ, НАХОДЯЩИХСЯ В ЭЛЕКТРИЧЕСКОМ ПОЛЕ (СТАЦИОНАРНОЕ СОСТОЯНИЕ)

Аннотация — В статье приводится методика оценки вклада диффузионного термоэффекта в суммарный перенос тепла в газовой смеси, находящейся в электрическом поле. Рассмотрен случай стационарного состояния.